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# N-FREE EXTENSIONS OF POSETS. NOTE ON A THEOREM OF P.A.GRILLET.

MAURICE POUZET\* AND NEJIB ZAGUIA

ABSTRACT. Let  $S_N(P)$  be the poset obtained by adding a dummy vertex on each diagonal edge of the  $N$ 's of a finite poset  $P$ . We show that  $S_N(S_N(P))$  is  $N$ -free. It follows that this poset is the smallest  $N$ -free barycentric subdivision of the diagram of  $P$ , poset whose existence was proved by P.A. Grillet. This is also the poset obtained by the algorithm starting with  $P_0 := P$  and consisting at step  $m$  of adding a dummy vertex on a diagonal edge of some  $N$  in  $P_m$ , proving that the result of this algorithm does not depend upon the particular choice of the diagonal edge chosen at each step. These results are linked to drawing of posets.

**Keywords:** posets, drawing,  $N$ -free posets, barycentric subdivision.

**AMS subject classification [2000]** Partially ordered sets and lattices (06A, 06B)

## 1. INTRODUCTION

An  $N$  is a poset made of four vertices labeled  $a, b, c, d$  such that  $a < c, b < c, b < d, b$  incomparable to  $a, a$  incomparable to  $d$  and  $d$  incomparable to  $c$  (see Figure 1(a)). This simple poset plays an important role in the algorithmic of posets [3]. It can be contained in a poset  $P$  in essentially two ways, leading to the characterization of two basic types of posets, the *series-parallel* posets and the *chain-antichain complete* (or C.A.C) posets.

The first way is related to the comparability graph of  $P$ . An  $N$  can be contained in  $P$  as an induced poset, that is  $P$  contains four vertices on which the comparabilities are those indicated above. Finite posets with no induced  $N$  are called *series-parallel*. Indeed, since their comparability graph contains no induced  $P_4$  (a four vertices path) they can be obtained from the one element poset by direct and complete sums (a result which goes back to Sumner [5], see also [6]). The second way is related to the (oriented) diagram of  $P$ . This is the object of this note.

In order to describe this other way, let us recall that a *covering pair* in a poset  $P$  is a pair  $(x, y)$  such that  $x < y$  and there is no  $z \in P$  such that  $x < z < y$ . The (directed) *diagram* of  $P$  is the directed graph, denoted by  $Diag(P)$ , whose vertex set is  $P$  and edges are the covering pairs of  $P$ . If  $(x, y)$  is a covering pair, we say that  $x$  is *covered by*  $y$ , or  $y$  *covers*  $x$ , a

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fact that we denote  $x \prec_P y$ , or  $x \prec y$  if there is no risk of confusion, or  $(x, y) \in \text{Diag}(P)$ . We denote by  $\text{Inc}(P)$  the set of pairs  $(x, y)$  formed of incomparable elements.

**Definition 1.1.** *Let  $a, b, c$ , and  $d$  four elements of  $P$ , we say that these elements form:*

- (1) *an  $N$  in  $P$  if  $b \prec c$ ,  $a \prec c$ ,  $b \prec d$ , and  $(a, d) \in \text{Inc}(P)$ ;*
- (2) *an  $N'$  in  $P$  if  $b \prec c$ ,  $a < c$ ,  $b < d$ , and  $(a, d) \in \text{Inc}(P)$ ;*
- (3) *an  $N$  in  $\text{Diag}(P)$  if  $b \prec c$ ,  $a \prec c$ ,  $b \prec d$ , and  $a \not\prec d$ ;*

An  $N$  in  $P$  is also an  $N'$  in  $P'$  and, provided that  $P$  is finite, an  $N'$  in  $P$  yields an  $N$   $\{a', b, c, d'\}$  in  $P$ . An  $N$  in  $P$  induces an  $N$  in  $\text{Diag}(P)$ ; the converse is false: if  $\{a, b, c, d\}$  is an  $N$  in  $\text{Diag}(P)$  as above, then  $a < d$  is possible, but -provided that  $P$  is finite- the 4 element subset  $a', a, c, b$ , where  $a \prec a' \leq d$ , is an  $N$  in  $P$ . Thus, if  $P$  is finite, it contains an  $N$  under one of these three forms if it contains all. We say that  $P$  is  $N$ -free if it contains no  $N$ . It was proved by P.A.Grillet [1] that *a finite poset  $P$  is  $N$ -free if and only if  $P$  is chain-antichain complete (or C.A.C) that is if every maximal chain of  $P$  meets every maximal antichain of  $P$*  (the formulation  $N$ -free in terms of the  $N$  defined in 1) is due to Leclerc and Monjardet [2]).

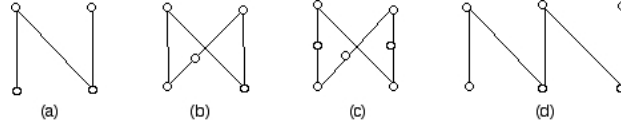


FIGURE 1. Examples of posets containing an  $N$ .

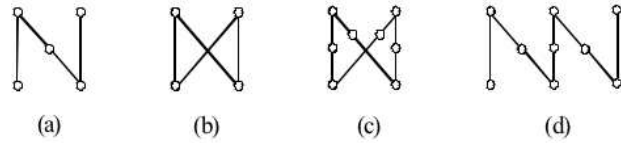


FIGURE 2. Examples of  $N$ -free posets

A *barycentric subdivision* of the diagram of a poset  $P$  consists to add finitely many vertices, possibly none, on each edge of the diagram of  $P$ . These vertices added to those of  $P$  provides a new poset in which  $P$  is embedded. We denote by  $S(P)$  the poset obtained by adding just one vertex on each edge of the diagram of  $P$ . As it is immediate to see, this poset is  $N$ -free. In his embedding theorem (Theorem 7 [1]) P.A.Grillet proves that among the  $N$ -free posets obtained as barycentric subdivisions of a finite poset  $P$  there is one, denoted  $\overline{P}$ , which is minimal. In this note, we provide a simple description of  $\overline{P}$  and give some consequences.

If  $A := \{a, b, c, d\}$  is an  $N$  in  $P$  (resp. a  $N$  in  $\text{Diag}(P)$ ), as in Definition 1.1, we say that the pair  $(b, c)$  is the *diagonal edge* of this  $N$ . Let  $N_{\text{diag}}(P)$  be the set of diagonal edges of all the  $N$ 's in  $P$  and let  $S_N(P)$  be the poset obtained by adding a dummy vertex on each edge in  $N_{\text{diag}}(P)$ .

**Theorem 1.2.** *Let  $P$  be a finite poset. Then  $S_N(S_N(P))$  is  $N$ -free. In fact this is the smallest  $N$ -free poset  $\overline{P}$  which comes from a barycentric subdivision of  $\text{Diag}(P)$ .*

This result translates to an algorithm which transforms a poset into an  $N$ -free poset: execute twice the algorithm consisting to add simultaneously a vertex on each  $N$  of a poset. Figure 3 shows an execution of this algorithm. Two dummy elements 6 and 7 are created during the first execution. Another two, 8 and 9, are produced during the second execution. After the second execution, the resulting poset does not contain an  $N$ . Instead of

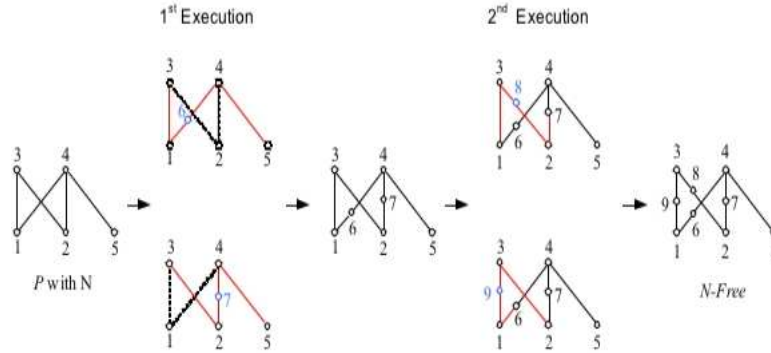


FIGURE 3. Execution of the algorithm

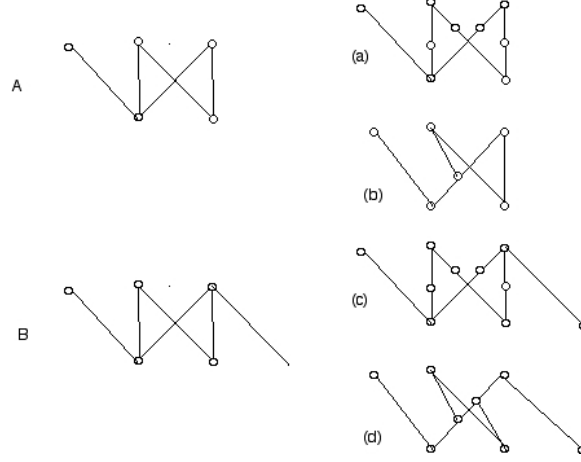
adding simultaneously the dummy vertices, we may add them successively.

**Theorem 1.3.** *The algorithm starting with  $P_0 := P$  and adding at step  $m$  a dummy vertex on a diagonal edge of some  $N$  in  $P_m$  stops on  $\overline{P}$ . Hence the result and the number of steps does not depends upon the particular choice of the diagonal edges choosen at each step.*

**Remarks 1.4.** (1) *If instead of the diagonal edges of  $P$  we consider those of  $\text{Diag}(P)$ , one get the same conclusion as in Theorem 1.2 and Theorem 1.3 (see Remark 2.6 below ).*

(2) *A poset  $P$  can be embedded into an  $N$ -free poset which does not come from a barycentric extension of its diagram, but a minimal one is not necessarily isomorphic to  $\overline{P}$  or to a quotient of  $\overline{P}$ . The posets represented in (b) and (d) of Figure 4 are minimal  $N$ -free extensions of  $A$  and  $B$ ; the first one is a quotient of  $\overline{A}$  represented in (a) the second is not a quotient of  $\overline{B}$  represented in (c).*

(3) *P.A. Grillet considered infinite posets satisfying some regularity condition. We restricted ourselves to finite posets. How our results translate to the infinite?*

FIGURE 4. Minimal  $N$ -free extensions

The motivation for this research came from drawing of posets. A good drawing solution that works for all posets is clearly out of reach. However, if every poset can be embedded into another with a particular structure, and at the same time these particular structures can be nicely drawn, then this can lead to an interesting approximation of general ordered set drawing. In [4] was presented an approach for drawing  $N$ -free posets. The algorithm, called *LR-drawing* (LR for left-right), consists of three steps: The first step is to convert  $P$  into an  $N$ -free poset  $Q$ . The second step is to apply the LR-drawing to  $Q$ . The third and last step is to retrieve  $P$  from the drawing of  $Q$ . The first part of the algorithm requiring to look at the possible extensions of a poset into an  $N$ -free one, this suggested an other look at the barycentric extensions of a poset and lead to the present results.

## 2. PROOFS

In this section, we consider a *finite poset*  $P$ . A basic ingredient of the proofs is the set  $A(P)$  of  $(b, c) \in \text{Diag}(P) \setminus N_{\text{diag}}(P)$  such that there are two vertices  $a, d \in P$  such that  $a < c, b < d$ ,  $(a, b), (c, d) \in \text{Inc}(P)$ , and either  $(a, c) \in N_{\text{diag}}(P)$  or  $(b, d) \in N_{\text{diag}}(P)$ . In our definition of members of  $A(P)$ , we could have supposed  $a \prec c$  and  $b \prec d$ . The definition we choose is closer to the one considered in Lemma 11 of Grillet's paper.

An important feature of a barycentric subdivision is that each new element has a unique upper cover and a unique lower cover. This fact is at the root of the following lemma.

**Lemma 2.1.** *Let  $P'$  be a barycentric subdivision of  $P$  and  $a, b, c, d \in P'$ . If  $a < c, b < d, (b, c) \in \text{Diag}(P')$  and  $(a, b), (d, c) \in \text{Inc}(P')$  then  $b, c \in P$ ; if, moreover,  $(a, c), (b, d) \in \text{Diag}(P')$  and  $a < d$  then  $a, d \in P$ .*

**Proof.** If  $b$  or  $c$  is not in  $P$  then  $(b, c)$  is a new edge, hence either  $b$ , or  $c$ , is a dummy vertex. If  $b$  is a dummy vertex, we have  $c < d$ , whereas

if  $c$  is a dummy vertex we have  $a < b$ , contradicting our hypothesis. If  $a \notin P$  then  $(a, c)$  is a new edge and thus  $a$  is a dummy vertex on some edge  $(a', c) \in \text{Diag}(P)$ ; from  $a < d$ , we get  $c < d$ , a contradiction. Applying this to the dual poset  $P^{\text{dual}}$  we get  $d \in P$ .  $\square$

**Lemma 2.2.** *Let  $\{a, b, c, d\}$  four elements of  $P$  such that  $(a, c), (b, d) \in \text{Diag}(P)$ ,  $(b, c) \in \text{Diag}(P) \setminus N_{\text{diag}}(P)$ .*

- (1)  $a < d$  and if  $(a, d) \notin \text{Diag}(P)$  then  $(a, c), (b, d) \in N_{\text{diag}}(P)$ ;
- (2) If  $(a, c) \in N_{\text{diag}}(P)$  then
  - (a)  $(x, b) \in \text{Inc}(P)$  for every  $x \in P$  such that  $(a, x) \in \text{Diag}(P)$  and  $\{a, c, x, y\}$  witnesses the fact that  $(a, c) \in N_{\text{diag}}(P)$  for some  $y \in P$ ;
  - (b)  $(a, d) \in N_{\text{diag}}(P)$  iff  $(a, d) \in \text{Diag}(P)$ .
- (3)  $(a, c) \in N_{\text{diag}}(P)$  if and only if there is some  $x \in P$  such that  $(a, x) \in \text{Diag}(P)$  and  $(x, b) \in \text{Inc}(P)$ .

**Proof.** (1) If  $a \not\prec d$  then  $\{a, b, c, d\}$  is an  $N$  in  $P$  hence  $(b, c) \in N_{\text{diag}}(P)$  contradicting the fact that  $(b, c) \in \text{Diag}(P) \setminus N_{\text{diag}}(P)$ . Let  $x \in P$  such that  $a \prec x \leq d$ . Then  $\{x, a, c, b\}$  is an  $N$  in  $P$  hence  $(a, c) \in N_{\text{diag}}(P)$ . With this argument applied to  $P^{\text{dual}}$  we get  $(b, d) \in N_{\text{diag}}(P)$ .

(2) Suppose  $(a, c) \in N_{\text{diag}}(P)$ . (i). Let  $x, y$  such that  $(a, x), (y, c) \in \text{Diag}(P)$  such that  $\{x, a, c, y\}$  witnesses that  $(a, c) \in N_{\text{diag}}(P)$ . If  $(x, b) \notin \text{Inc}(P)$  then  $b < x$ . Let  $b' \in P$  such that  $b \prec b' \leq x$ . Then  $\{y, c, b', d\}$  is an  $N$  in  $P$  thus  $(b, c) \in N_{\text{diag}}(P)$  contradicting our hypothesis. (ii). Suppose  $(a, d) \in \text{Diag}(P)$ . Let  $x, y$  as above. Since  $(x, b) \in \text{Inc}(P)$ ,  $\{x, a, d, b\}$  is an  $N$  in  $P$ , hence  $(a, d) \in N_{\text{diag}}(P)$ . The converse is obvious.

(3) follows immediately from (2 – a).  $\square$

**Lemma 2.3.**  $N_{\text{diag}}(S_N(P)) = A(P)$

**Proof.** Set  $P' := S_N(P)$ .

(a)  $N_{\text{diag}}(P') \subseteq A(P)$ . Let  $(b, c) \in N_{\text{diag}}(P')$ .

**Claim 1**  $(b, c) \in \text{Diag}(P) \setminus N_{\text{diag}}(P)$ . Moreover, if  $A := \{a, b, c, d\}$  is an  $N$  in  $P'$  with  $a \prec_{P'} c$  and  $b \prec_{P'} d$  then  $a$  or  $d$  are in  $P' \setminus P$ .

**Proof of Claim 1** According to Lemma 2.1 we have  $b, c \in P$ . Since  $(b, c) \in \text{Diag}(P')$ , it follows  $(b, c) \in \text{Diag}(P) \setminus N_{\text{diag}}(P)$ . Since  $b, c \in P$ , if  $a$  and  $d$  are in  $P$  then  $\{a, b, c, d\}$  is an  $N$  in  $P$  and thus  $(b, c)$  has been subdivided, hence  $(b, c) \notin \text{Diag}(P')$  a contradiction.  $\square$

Let  $A$  as above.

**Case 1.**  $a \in P' \setminus P$ . In this case  $a$  is a dummy vertex on some edge  $(a', c) \in N_{\text{diag}}(P)$ . Since  $(b, d) \in \text{Diag}(P')$  there is some  $d' \in P$  such that  $b \prec_P d'$  and  $d \leq d'$  ( $d' = d$  if  $d \in P$ , otherwise  $(b, d) \in \text{Diag}(P')$  in which case  $d$  is a dummy vertex on  $(b, d')$ ). Thus  $A' := \{a', b, c, d'\}$  witnesses the fact that  $(b, c) \in A(P)$ .

**Case 2.**  $d \in P' \setminus P$ . This case reduces to Case (1) above by considering the dual poset  $P^{\text{dual}}$ . From Claim 1 there is no other case. The proof of (a) is complete.

(b)  $A(P) \subseteq N_{diag}(P')$ . Let  $(b, c) \in A(P)$ . Let  $\{a, b, c, d\}$ , with  $(a, c), (b, d) \in Diag(P)$ , witnessing it. If  $(a, c) \in N_{diag}(P)$ , let  $u$  be a dummy vertex on  $(a, c)$  then  $\{u, c, b, d'\}$ , where  $d' := d$  if  $(b, d) \notin N_{diag}(P)$  and  $d'$  is a dummy vertex on  $(b, d)$  otherwise, is an  $N$  in  $P'$  hence  $(b, c) \in N_{diag}(P')$ . If  $(b, d) \in N_{diag}(P)$ , apply the above case to  $P^{dual}$ .  $\square$

**Lemma 2.4.**  $A(S_N(P)) = \emptyset$

**Proof.** Suppose the contrary. Set  $P' := S_N(P)$  and let  $(b, c) \in A(P')$ . Let  $A := \{a, b, c, d\}$ , with  $(a, c), (b, d) \in Diag(P')$ , witnessing the fact that  $(b, c) \in A(P')$ . According to (1) of Lemma 2.2 applied to  $P'$ , we have  $a < d$ . Thus from Lemma 2.1, we have  $a, b, c, d \in P$ .

**Case 1.**  $(a, c) \in N_{diag}(P')$ . According to (3) of Lemma 2.2 applied to  $P'$  there is some  $x \in P'$  such that  $(a, x) \in Diag(P')$  and  $(x, b) \in Inc(P')$ .

Next,  $x \in P' \setminus P$ . Indeed,  $\{x, a, c, b\}$  is an  $N$  in  $P'$ . Thus, if  $x \in P$ , this is an  $N$  in  $P$  and  $(a, c) \in N_{diag}(P)$ , hence a dummy vertex is added on  $(a, c)$  in  $P'$  contradicting  $(a, c) \in Diag(P')$ . Finally, we consider two subcases:

**Subcase 1.1.**  $(a, d) \in Diag(P')$ . In this case,  $(x, d) \in Inc(P)$  and, since  $x \notin P$ ,  $(a, x) \in Diag(P') \setminus Diag(P)$ . Hence, there is  $x' \in P$  such that  $x$  is a dummy vertex of  $(a, x') \in N_{diag}(P)$ . Let  $A' := \{x', a, c, b\}$ . We have  $(a, c), (b, c) \in Diag(P) \setminus N_{diag}(P)$  and  $(a, x') \in N_{diag}(P)$ . Thus  $(a, c) \in A(P)$ . According to (1) of Lemma 2.2  $(b, x') \in Diag(P)$ . Next, according to (3) of Lemma 2.2, there is some  $v \in P$  such that  $(v, x') \in Diag(P)$  and  $(v, c) \in Inc(P)$ . It follows that  $\{v, x', b, c\}$  is an  $N$  in  $P$  hence  $(b, x') \in N_{diag}(P)$ . If  $b'$  is a dummy vertex on  $(b, x')$  then  $\{b', b, c, a\}$  is an  $N$  in  $P'$  hence  $(b, c) \in N_{diag}(P')$  contradicting  $(b, c) \in A(P')$ . Thus this subcase leads to a contradiction.

**Subcase 2.2.**  $(a, d) \notin Diag(P')$ . In this case, we may suppose  $x < d$ . In fact  $(x, d) \in Diag(P')$ . Indeed, if  $(x, d) \notin Diag(P')$  then there is  $d' \in P$  such that  $x <_{P'} d' <_P d$ . But, then  $\{d', d, c, b\}$  is an  $N$  in  $P$ , thus  $(a, c) \in N_{diag}(P)$  proving that  $(a, c) \notin Diag(P')$  a contradiction. It follows that  $x$  is a dummy vertex added on  $(a, d) \in N_{diag}(P)$ . Since  $(a, d) \in N_{diag}(P)$ ,  $(b, d) \in Diag(P) \setminus N_{diag}(P)$  and  $(b, c) \in Diag(P)$ ,  $(b, d) \in A(P)$ . Since  $(a, c) \in Diag(P)$  it follows from (2–b) of Lemma 2.2 that  $(a, c) \in N_{diag}(P)$  contradicting  $(a, c) \in Diag(P')$ . This subcase leads to a contradiction too.

**Case 2.**  $(b, d) \in N_{diag}(P')$ . This case reduces to the previous one by considering the dual poset  $P^{dual}$ . Hence, it leads to a contradiction.

Consequently  $A(P') = \emptyset$ . The proof is complete.  $\square$

**Proof of Theorem 1.2.** Let  $P' := S_N(S_N(P))$ . According to Lemma 2.3 and Lemma 2,  $N_{diag}(P') = A(S_N(P)) = \emptyset$ . Clearly, an  $N$ -free poset  $Q$  associated with a barycentric subdivision of  $Diag(P)$  must include  $S_N(P)$ . This applied to  $S_N(P)$  gives that  $Q$  contains  $P'$ . Hence  $P'$  is the smallest  $N$ -free poset coming from a barycentric subdivision of  $Diag(P)$ , thus it coincides with the poset  $\overline{P}$  constructed by P.A.Grillet.  $\square$

**Lemma 2.5.** Let  $P'$  with  $P \subseteq P' \subseteq S_N(S_N(P))$ ; then  $N_{diag}(P') \subseteq N_{diag}(P) \cup A(P)$ .

**Proof.** Let  $(b, c) \in N_{diag}(P')$ . Suppose  $(b, c) \notin N_{diag}(P) \cup A(P)$ . Let  $Q := S_N(S_N(P))$ . We claim that  $(b, c) \in N_{diag}(Q)$ . Let  $A := \{a, b, c, d\}$  be an  $N$  of  $P'$  witnessing the fact that  $(b, c) \in N_{diag}(P')$ . Since, from Lemma 2.3  $(b, c) \notin N_{diag}(P) \cup N_{diag}(S_N(P))$ ,  $(b, c) \in Diag(Q)$  thus  $A' := \{a', b, c, d'\}$  where  $a \leq a' \prec_Q c$  and  $b \prec_Q d'$  is an  $N$  in  $Q$  proving our claim. Next, with  $(b, c) \in N_{diag}(Q)$  and  $Q := S_N(S_N(P))$ , we get from Lemma 2.3 that  $(b, c) \in A(S_N(P))$ . Since, from Lemma ,  $A(S_N(P)) = \emptyset$ , we get a contradiction. This proves the lemma.  $\square$

**Proof of Theorem 1.3** An immediate induction using Lemma 2.5 shows that each  $P_m$  is a subset of  $Q := S_N(S_N(P))$ . Since  $Q$  is the least  $N$ -free subset of  $S(P)$  containing  $P$  the algorithm stops on  $Q$ . The number of steps is the size of  $N_{diag}(P) \cup A(P)$ .  $\square$

**Remarks 2.6.** (1) Let  $ND_{diag}(P)$  be the set of diagonal edges of the  $N$ 's in  $D(P)$ . Clearly,  $ND(P) \subseteq N_{diag}(P) \cup A(P)$ . Thus, with the same proof as for Theorem 1.3, we obtain that the algorithm consisting to add at step  $m$  a dummy vertex on an edge of some  $N$  in  $D(P_m)$  ends on  $\overline{P}$ . Similarly, with Lemma 2.5 we get that  $ND_{diag}(ND_{diag}(P)) = \overline{P}$ ;  
 (2) The fact that the algorithm given in Theorem 1.3 stops is obvious: at each step,  $P_m$  is a subset of  $S(P)$ . The fact that the number of steps is independent of the choosen edges is more significant. This suggests a deepest investigation. We just note that if  $P_m$  contains just one  $N$  then  $P_{m+1}$  is  $N$ -free (we leave the proof to the reader).

## REFERENCES

- [1] Grillet, P. A., Maximal chains and antichains, Fund. Math. **65**, (1969), 157–167.
- [2] Leclerc, B. and Monjardet, B., Ordres “C. A. C.”, Fund. Math. **79**, (1973) 11–22.
- [3] Rival, Ivan, Stories about order and the letter  $N$  (en). Combinatorics and ordered sets (Arcata, Calif., 1985), 263–285, Contemp. Math., 57, Amer. Math. Soc., Providence, RI, 1986.
- [4] Livaniaina Hary Rakotomalala, Guy-Vincent Jourdan, Nejib Zaguia, LR-drawing of  $N$ -free ordered sets, submitted;
- [5] Sumner, David P., Graphs indecomposable with respect to the  $X$ -join, Discrete Math., **6** (1973), 281–298.
- [6] Valdes, Jacobo; Tarjan, Robert E.; Lawler, Eugene L., The recognition of series parallel digraphs. SIAM J. Comput. 11 (1982), no. 2, 298–313.

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